The Demand and Supply of Safe Assets (Premilinary)

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Abstract

It is documented that over the past 60 years, the safe assets as a percentage share of total assets in the U.S. has been almost constant and fluctuates around 33%. Empirically, I find that the demand curve of safe assets as a share of total assets is stable at an almost constant share over time, while the supply curve of safe assets as a share of total assets fluctuates over the business cycle. I then present a model to show that a stable demand for safe assets combined with increased risk in asset returns can pull the economy into a liquidity trap with low output, low investment and low inflation.

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1 Introduction

Safe assets, or assets that are valued at face value without expensive and prolonged analysis, play a critical role in the economy (Gorton 2017). Gorton, Lewellen and Metrick (2012) documents that over the past 60 years, the safe assets as a percentage share of total assets has been almost constant and fluctuates around 33%. This paper aims to answer the following questions: What does this stable safe-asset share imply about the demand curve and supply curve of safe assets as a share of total assets? Can a stable demand for safe assets and increased risk in asset returns help explain the macroeconomic consequences of the great recession?

The first contribution of this paper is empirical. In particular, I show that the demand curve of safe assets as a share of total assets is stable at an almost constant share in the U.S. economy, while the supply curve of safe assets as a share of total assets fluctuates over the business cycles. I divide assets into two categories: safe assets and risky assets. The identification strategy I adopt is as follows: One of the scenarios in Figure 1 and Figure 2 must be true. In both figures, The demand (supply) of safe assets relative to risky assets, $lnQ_{safe} - lnQ_{risky}$, is decreasing (increasing) in the risk-adjusted expected return of risky assets relative to safe assets, $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. In Figure 1, demand of safe assets relative to risky assets is almost constant / vertical and supply fluctuates around. In this case, there will be negative co-movements in $lnQ_{safe} - lnQ_{risky}$ and $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. In Figure 2, supply of safe assets relative to risky assets is almost constant / vertical and demand fluctuates around. In this case, there will be positive co-movements in $lnQ_{safe} - lnQ_{risky}$ and $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$.

We construct quarterly measures of the quantity of safe assets Q_{safe} and quantity of risky assets Q_{risky} from the Federal Reserve's Flow of Funds database, following the method in Gorton, Lewellen and Metrick (2012). We do not observe the risk-adjusted expected return of risky assets relative to safe assets, $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$, so we will use real GDP growth rate and S&P 500 Index realized volatility as proxies for $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. Using the proxies, we find that $lnQ_{safe} - lnQ_{risky}$ is negatively correlated with $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. This result supports the case of Figure 1. Therefore, we conclude that the demand curve of safe assets as a share of total assets is stable at an almost constant share, while the supply curve of safe assets as a share of total assets fluctuates over the business cycles.

The second contribution of this paper is theoretical. In particular, I present a model to show that a stable demand for safe assets combined with increased risk in asset returns can pull the economy into a liquidity trap with low output, low investment and low inflation. The model is an overlapping generation model based on Eggertsson and Mehrotra (2014). In the model, the investors and lenders are highly risk averse. Increased risk in asset returns makes the lenders less willing to lend, hence reduce the spending of the borrowers and reduce aggregate demand. The fall in aggregate demand can pull the economy into a liquidity trap with low inflation. Faced with such a liquidity trap, the investors optimally choose to hold cash rather than investing in risky projects, which further deters aggregate demand. Therefore, the economy falls into a liquidity trap with low output, low investment and low inflation.

Related Literature

The paper is closely related to the literature that studies the role of safe assets in the economy (Cabellero 2006, Cabellero, Farhi and Gourinchas 2008, Bernanke, Bertaut, DeMarco and Kamin 2011, Gorton 2017). The most relevant paper is Gorton, Lewellen and Metrick (2012), where the authors document that the safe-asset share in the U.S. economy is stable over time. The main contribution of this paper is to show that it is the demand curve of safe assets as a share of total assets that is stable at an almost constant share over time, while the supply curve of safe assets as a share of total assets fluctuates over the business cycles.

The paper is closely related to the literature on liquidity trap (Eggertsson and Woodford 2004, Christiano, Eichenbaum and Rebelo 2011, Eggertsson and Krugman 2012). The model in the paper is based on the overlapping generation framework of Eggertsson and Mehrotra (2014) and is able to generate a permanent liquidity trap. The contribution of this paper is to show that a stable demand for safe assets combined with increased risk in asset returns can pull the economy into a liquidity trap with low output, low investment and low inflation. Caballero and Farhi (2017) shows that an insufficient supply of safe assets can pull the economy into a liquidity trap. This paper produces similar results as in their paper but offers an alternative micro-foundation.

Figure 1: Stable Demand of Safe-Asset Share, with Fluctuating Supply

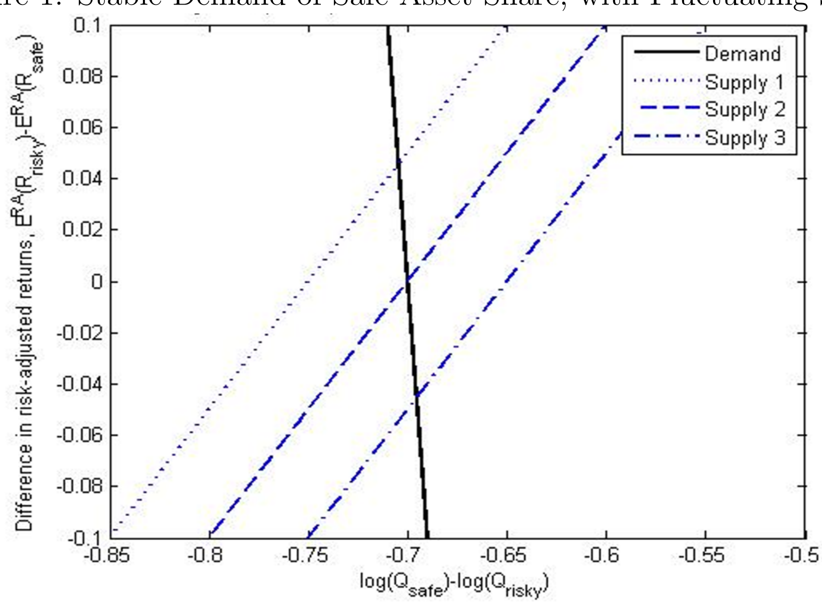
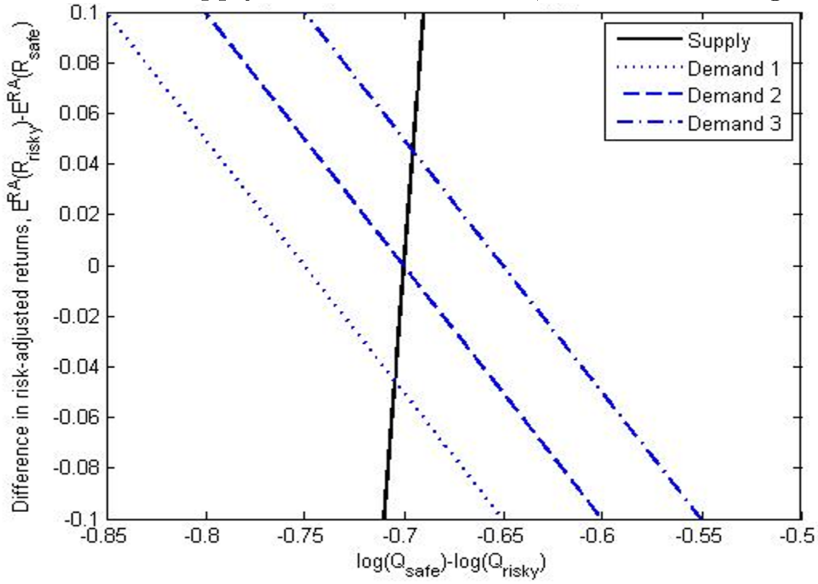


Figure 2: Stable Supply of Safe-Asset Share, with Fluctuating Demand



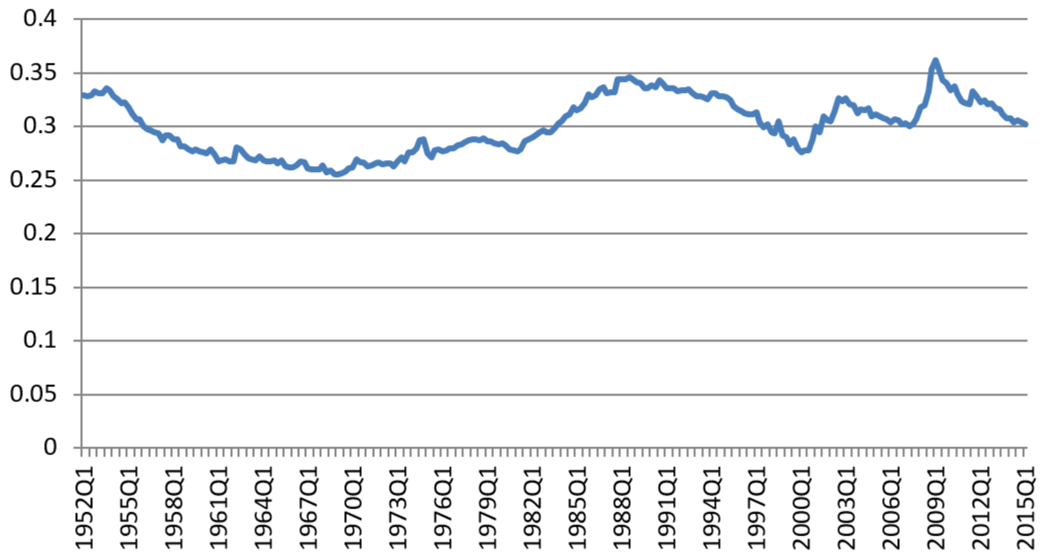
2 Empirical Evidence

Gorton, Lewellen and Metrick (2012) documents that over the past 60 years, the safe assets as a percentage share of the total assets has been almost constant and fluctuates around 33%. The safe assets include assets that are either directly or indirectly used in an information-insensitive fashion. I compute the quarterly safe-asset share from Federal Reserve's Flow of Funds database, following exactly the same method in Gorton, Lewellen and Metrick (2012). In particular, I use total liabilities and equity in all sectors as the measure of total assets. The key components of safe assets include bank deposits, money market mutual funds shares, commercial papers, federal funds and repurchase agreements, short-term interbank loans, treasuries, agency debt, municipal bonds, securitized debt, and high-grade financial-sector corporate debt. Assets that are not categorized as safe assets are categorized as risky assets.

Figure 3 plots the safe-asset share for each quarter over the past 60 years. We see that the safe assets as a percentage share of total assets is almost constant and fluctuates around 30%. Figure 3 matches reasonably well with the same figure in Gorton, Lewellen and Metrick (2012).

Given that the safe asset share is relatively stable over a long period of time, our identification strategy is to assume that one of the following scenarios should be true: either the supply curve of safe assets as a share of total assets is stable at an almost constant share over time, or the demand curve of the safe assets as a share of total assets is stable at an almost constant share over time. I illustrate the two possible scenarios in Figure 1 and Figure 2. I denote quantity (in dollar term) of the safe assets as Q_{safe}, quantity (in dollar term) of the risky assets as Q_{risky}, risk-adjusted expected return of risky

Figure 3: The safe asset share over time: 1952Q1 to 2015Q1



assets as $E^{RA}(R_{risky})$, and risk-adjusted expected return of safe assets as $E^{RA}(R_{safe})$. In both figures, the demand (supply) of safe assets relative to risky assets, $lnQ_{safe} - lnQ_{risky}$, is decreasing (increasing) in the risk-adjusted expected return of the risky assets relative to safe assets, $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. Intuitively, when risky assets offer a higher risk-adjusted expected return relative to safe assets, the demand of safe assets relative to risky assets will decrease and the supply of safe assets relative to risky assets will increase.

In Figure 1, demand of safe assets as a share of total assets is almost constant / vertical, and supply fluctuates over the business cycle. In this case, $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$ and $lnQ_{safe} - lnQ_{risky}$ will move in the opposite directions. In Figure 2, supply of safe asset as a share of total assets is almost constant / vertical, and demand fluctuates over the business cycle. In this case, $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$ and $lnQ_{safe} - lnQ_{risky}$ will move in the same direction. Therefore, it suffices to test empirically whether $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$ and $lnQ_{safe} - lnQ_{risky}$ have positive or negative co-movements over the business cycle.

There remains one more difficulty: we don't have any direct measure for the risk-adjusted expected returns of risky assets relative to safe assets, $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. I will use two variables, real GDP growth rate and S&P 500 Index realized volatility, as proxies for $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. The proxies and their co-movements with $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$ are listed in in Table 1. First, I assume higher (lower) real GDP growth rate corresponds with higher (lower) $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. Intuitively, during periods of faster economic growth people are more inclined to take on risky investments, so the risk-adjusted expected return of risky assets relative to safe assets is higher. Second, I assume higher (lower) S&P 500 Index realized volatility corresponds with lower (higher) $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$. Intuitively, higher stock index realized volatility indicates more risk and uncertainty in the economy, so the risk-adjusted expected return of

Table 1: Proxies and Their Co-movements with $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$

Real GDP growth rate	positive co-movements
S&P 500 Index realized volatility	negative co-movements

Table 2: The Proxies' Correlations with $lnQ_{safe} - lnQ_{risky}$

Real GDP growth rate	-0.17
S&P 500 Index realized volatility	0.30

risky assets relative to safe assets is lower.

Table 2 shows the proxies' correlations with the $lnQ_{safe} - lnQ_{risky}$. Combining with Table 1, we see that for real GDP growth rate, which tends to move in the same direction with $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$, the correlation with $lnQ_{safe} - lnQ_{risky}$ is negative. For S&P 500 Index realized volatility, which tends to move in opposite directions with $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$, the correlation with $lnQ_{safe} - lnQ_{risky}$ is positive. Such evidences indicate that $E^{RA}(R_{risky}) - E^{RA}(R_{safe})$ is negatively correlated with $lnQ_{safe} - lnQ_{risky}$, and hence support the case of Figure 1.[1] Therefore, we conclude that the demand curve of safe assets as a share of total assets is stable at an almost constant share, while the supply curve of safe assets as a share of total assets fluctuates over the business cycles.

3 Model

In this section, I present a theoretical model to show that a stable demand for safe assets combined with increased risk in asset returns can pull the economy into a liquidity trap with low output, low investment and low inflation. The model is based on Eggertsson and Mehrotra (2014). Consider an overlapping generation economy, where an agent live for three periods: Young, Middle-aged, and Old. The utility of the agent is

$$u(C_t^y, C_{t+1}^m, C_{t+2}^o) = log(C_t^y) + \beta log(C_{t+1}^m) + \beta^2 min\{log(C_{t+2}^o)\} \quad (1)$$

This utility set-up means the middle-aged agent is infinitely risk averse, so that for a middle-aged agent, he will care only about the minimum / safe return of any investment he makes. As will be clear later, the middle-aged agent will the investor and lender in the economy. The empirical evidence provided in Section 2 suggests that such modeling choice is reasonable – there is always a stable demand of safe assets in the economy.

[1]The results remain the same if we use lagged real GDP growth rate in the previous quarter and lagged S&P 500 Index realized volatility in the previous quarter as proxies.

A representative firm in this economy has the following technology:

$$Y_S = \begin{cases} L^\alpha, & \text{with probability p} \\ min\{D, L^\alpha\}, & \text{with probability } 1-p \end{cases} \tag{2}$$

With probability $1-p$, the firm may suffer a bad shock and produce a low output D. We send the probability of $1-p$ close to zero, so the bad state will almost never materialize. However, since the middle-aged agent is infinitely risk averse, even the slightest chance of a bad state will matter when the middle-aged agent makes his decisions.

Only the middle-aged agent will work in the firm, and the firm's entire profits are distributed to the middle-aged agent only. The middle aged agent is endowed with $\bar{L}$ unit of labor. The young agent can borrow from the middle-aged agent to smooth consumption. However, because the lender (the middle-aged agent) is infinitely risk averse, he will not lend more than the young agent's safe income next period, which is D according to (2).

The young agent is endowed with 1 unit of projects. There are two kinds of projects that a young agent can operate: safe projects and risky projects. The young agent is endowed with s $(0 < s < 1)$ unit of safe projects and $1-s$ unit of risky projects. Each unit of safe projects will produce 1 unit of consumption goods next period. Each unit of risky projects will produce 1 unit of consumption goods next period with probability p, and $b < 1$ unit of consumption goods with probability $1-p$. As before, we send $1-p$ close to zero, so that the bad state will almost never materialize. The only way the young agent can start the projects is by selling the entire future proceeds of the projects to the middle-aged agent. The young agent will not be able to start the projects without such investment from the middle-aged agent.

I impose the assumption that the young agent will never sell any of his project for less than h units of consumption goods - the intuition behind this assumption is that it is costly for the young agent to start this project so he wants enough goods in return. For the middle-aged, since they are infinitely risk-averse, they care only about the lowest payoff of the projects, which is 1 for the safe projects and $b < 1$ for the risky projects. Hence if the risky project is sold at the minimum possible price h, the safe real return for the middle-aged agent is b/h.

The minimum output from the representative firm, D, and the amount of safe projects that can be operated by the young agent, s, measure the riskiness of asset returns in the economy. In reality, many factors could affect D and s. Changes in D and s can be caused by changes in the expectation of asset returns, especially in the tail risk involved in the assets. Changes in D and s can also be caused by dis-functioning financial markets that are unable to effectively diversify risk.

3.1 Aggregate Demand

We denote the real interest rate as r_t and the nominal interest rate as i_t. The risky project will not be carried out if

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}} > \frac{b}{h} \tag{3}$$

The condition in (3) says the risky projects will be not carried out if the real return of saving money in bank accounts, $(1 + i_t)\frac{P_t}{P_{t+1}}$, is higher than the minimum safe return of the risky project, $\frac{b}{h}$. In that case, the middle-aged agent will choose to save money in bank accounts rather than investing in risky projects. Since the safe projects offer higher safe return than the risky projects, we assume that the safe project will always be carried out.

The consumption of the young agent is constrained by the following condition

$$C_t^y \leq \frac{D}{1 + r_t} + \frac{s}{1 + r_t} + \frac{b(1 - s)I_{\{1+r_t \leq \frac{b}{h}\}}}{1 + r_t} \tag{4}$$

The first term on the right hand side of (4) represents the borrowing against future safe income D. The second term on the right hand side of (4) represents the proceeds the young agent makes from selling the safe projects to the middle-aged agent. The third term on the right hand side of (4) represents the proceeds the young agent makes from selling the risky projects to the middle-aged agent. As discussed before, the risky projects will be carried out only when $1 + r_t \leq b/h$, and this is captured by the indicator term $I_{\{1+r_t \leq b/h\}}$.

The utility maximization problem of the agent is:

$$\max_{\{C_t^y, C_{t+1}^m, C_{t+2}^o\}} log(C_t^y) + \beta log(C_{t+1}^m) + \beta^2 min\{log(C_{t+2}^o)\} \tag{5}$$

$$\begin{aligned} s.t. \quad & C_t^y = B_t^y \\ & B_t^y \leq \frac{D}{1 + r_t} + \frac{s}{1 + r_t} + \frac{b(1 - s)I_{\{1+r_t \leq \frac{b}{h}\}}}{1 + r_t} \\ & C_{t+1}^m = Y_{t+1} - B_t^y + B_{t+1}^m \\ & C_{t+2}^o = (1 + r_{t+1})B_{t+1}^m \end{aligned}$$

Here B_t^y is the amount the young borrowed from the middle-aged plus the proceeds he makes from selling his projects. B_{t+1}^m is the saving of the middle-aged agent.

In the following analysis, I assume that the constraint (4) for the young agent always binds, so $B_t^y = \frac{D}{1+r_t} + \frac{s}{1+r_t} + \frac{bI_{\{1+r_t < \frac{b}{h}\}}}{1+r_t}$. Solving the utility maximization problem in (5), we have:

$$C_t^y = \frac{D}{1 + r_t} + \frac{s}{1 + r_t} + \frac{b(1 - s)I_{\{1+r_t \leq \frac{b}{h}\}}}{1 + r_t}$$

$$C^m_{t+1} = \frac{1}{1+\beta}\big(Y_{t+1} - D - s - (1-s)I_{\{1+r_t \le \frac{b}{h}\}}\big)$$

$$C^o_{t+2} = \frac{\beta(1+r_{t+1})}{1+\beta}\big(Y_{t+1} - D - s - (1-s)I_{\{1+r_t \le \frac{b}{h}\}}\big)$$

We solve for the real interest rate by equating loan demand L^d_t with loan supply L^s_t.

$$L^d_t = \frac{D + s + b(1-s)I_{\{1+r_t \le \frac{b}{h}\}}}{1+r_t}$$

$$L^s_t = \frac{\beta}{1+\beta}\big(Y_t - D - s - (1-s)I_{\{1+r_t \le \frac{b}{h}\}}\big)$$

Imposing the loan market clearing condition gives us

$$1 + r_t = \frac{1+\beta}{\beta}\frac{D + s + b(1-s)I_{\{1+r_t \le \frac{b}{h}\}}}{Y_t - D - s - (1-s)I_{\{1+r_t \le \frac{b}{h}\}}} \tag{6}$$

We further assumes that the central bank follows a policy rule:

$$1 + i_t = \max\{1, (1+i^*)(\frac{\Pi_t}{\Pi^*})^{\phi_\pi}\} \tag{7}$$

Here $\phi_\pi > 1$, so we have nominal interest rate move more than one for one with respect to inflation Π_t. This policy rule also suggests that at or below the inflation level $\Pi_{kink} = \big(\frac{1}{1+i^*}\big)^{\frac{1}{\phi_\pi}}\Pi^*$, nominal interest rate i_t will hit the zero lower bound.

The no arbitrage condition requires

$$1 + r_t = (1+i_t)\frac{P_t}{P_{t+1}} \tag{8}$$

We will focus on the steady state equilibrium of the economy. Combining equation (6), (7) and (8), we have the following aggregate demand (AD) curve in the steady state.

$$Y = \begin{cases} D + s + \frac{1+\beta}{\beta}\frac{(D+s)}{1+i^*}\frac{\Pi*^{\phi_\pi}}{\Pi^{\phi_\pi - 1}}, & \text{for } \Pi \ge \Pi_{kink} \text{ and } \Pi > (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}}\Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}} \\ D + 1 + \frac{1+\beta}{\beta}\frac{(D+s+b(1-s))}{1+i^*}\frac{\Pi*^{\phi_\pi}}{\Pi^{\phi_\pi - 1}}, & \text{for } \Pi \ge \Pi_{kink} \text{ and } \Pi \le (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}}\Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}} \\ D + 1 + \frac{1+\beta}{\beta}(D + s + b(1-s))\Pi, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi \ge \frac{h}{b} \\ D + s + \frac{1+\beta}{\beta}(D+s)\Pi, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi < \frac{h}{b} \end{cases} \tag{9}$$

The AD curve is defined in four pieces. Intuitively, when $i = 0$ ($\Pi < \Pi_{kink}$) and inflation is too low ($\Pi < \frac{h}{b}$), the real return of holding money is high. Hence the middle-aged agent will choose to simply hold money rather than investing in the risky projects operated by the young agent. When $i > 0$ ($\Pi > \Pi_{kink}$), nominal rate moves more than one for one with respect to inflation, so when inflation is high enough ($\Pi > (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}}$), nominal interest rate will be even higher and so is the real return of saving money in bank accounts. Hence the middle-aged agent will again choose to save in the bank accounts rather than investing in the risky projects operated by the young agent. When inflation is moderate ($\Pi \geq \frac{h}{b}$ and $\Pi \leq (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}}$), the middle-aged agent will invest in the risky projects operated by the young agent, so an additional $1 - s$ units of output from the risky projects will be produced and consumed.

3.2 Aggregate Supply

I follow Eggertsson and Mehrotra (2014) and impose nominal wage rigidity to derive the aggregate supply (AS) curve. The firm maximize profit Z_t

$$Z_t = \max_{L_t} P_t L_t^\alpha - W_t L_t \tag{10}$$

The first order condition gives

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha - 1} \tag{11}$$

We assume the following wage rigidity

$$W_t = \max\{\tilde{W}_t, P_t \alpha \bar{L}^{\alpha-1}\}, \quad \tilde{W}_t = \gamma W_{t-1} + (1-\gamma) P_t \alpha \bar{L}^{\alpha-1} \tag{12}$$

Here $P_t \alpha \bar{L}^{\alpha-1}$ is the wage at full employment level $\bar{L}$. Such wage rigidity in (12) means the wage cannot fall below certain threshold $\tilde{W}_t$. If $\gamma = 1$, nominal wage cannot be adjusted downwards. If $\gamma = 0$, nominal wage is perfectly flexible. When $\Pi > 1$, nominal wage tends to increase and nominal wage rigidity does not matter. However, when $\Pi < 1$, nominal wage will tend to decrease, and nominal wage rigidity will play a role here. When the nominal wage rigidity constraint binds, from (12) we have

$$\frac{W_t}{P_t} = \gamma \frac{W_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + (1-\gamma)\alpha \bar{L}^{\alpha-1} \tag{13}$$

We will again focus on the steady state of the economy. From equation (13), the steady state real wage can be written as

$$w = \frac{(1-\gamma)\alpha \bar{L}^{\alpha-1}}{1 - \gamma \Pi^{-1}} \tag{14}$$

Combining equation (11) with equation (14) gives us

$$Y_S = \begin{cases} \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f, & \text{for } \pi < 1 \\ Y^f, & \text{for } \pi > 1 \end{cases} \tag{15}$$

Here $Y^f = \bar{L}^\alpha$ is the output level of the representative firm at full employment. Notice that in addition to the output produced by the representative firm, we also need to take into account the output produced by the projects operated by the young agent. As a result, the aggregate supply (AS) curve in the steady state is

$$Y = \begin{cases} Y_S + s, & \text{for } \Pi \geq \Pi_{kink} \text{ and } \Pi > (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}} \\ Y_S + 1, & \text{for } \Pi \geq \Pi_{kink} \text{ and } \Pi \leq (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}} \\ Y_S + 1, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi \geq \frac{h}{b} \\ Y_S + s, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi < \frac{h}{b} \end{cases} \tag{16}$$

We see the AS curve is defined in four pieces. Intuitively, when inflation gets too high ($\Pi \geq \Pi_{kink}$ and $\Pi > (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}}$) or too low ($\Pi < \Pi_{kink}$ and $\Pi < \frac{h}{b}$), the return of saving in bank accounts surpasses the safe return of investing in the risky projects operated by the young agent, so the middle-aged agent will not invest in the risky projects operated by the young. The $1 - s$ units of output of the risky projects are hence lost in those cases.

3.3 Equilibrium

The equilibrium is defined as a set of allocations and prices such that consumers maximize utility as in (5), firms maximize profits as in (10), wages are sticky as in (12), monetary policy follows policy rule as in (7), and all markets clear. The steady state equilibrium can be fully characterized by the AD curve in (9) and the AS curve in (16).

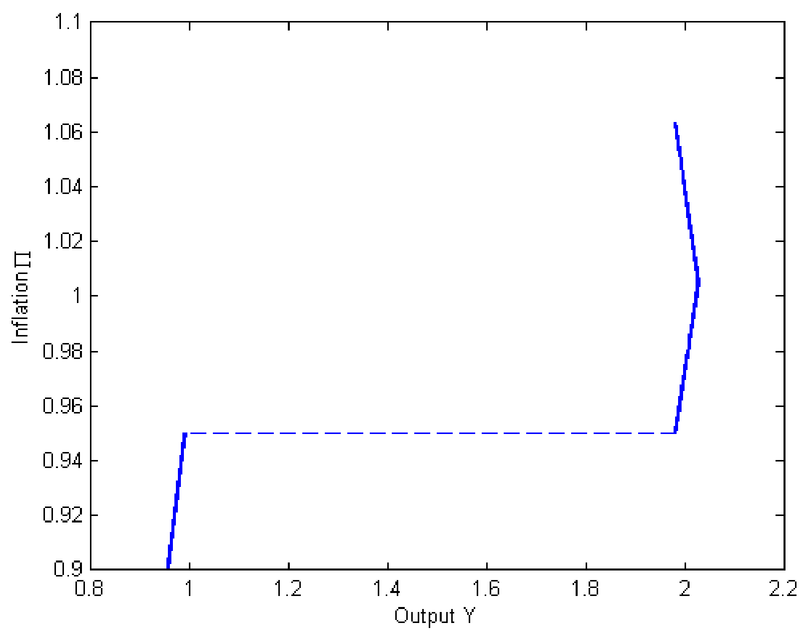
After the financial crisis, the economy falls into a state with low inflation, low output, low investment and zero nominal interest rate. We are hence more interested in such cases in the model. In the numerical analysis to follow, we will not worry about the case when inflation is too high ($\Pi > \Pi_{kink}$ and $\Pi > (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}}$), and focus on the other three cases where inflation is not too high. Throwing away the high inflation case, the truncated AD and AS curves are:

$$Y = \begin{cases} D + 1 + \frac{1+\beta}{\beta} \frac{(D+s+b(1-s))}{1+i^*} \frac{\Pi *^{\phi_\pi}}{\Pi^{\phi_\pi - 1}}, & \text{for } \Pi \geq \Pi_{kink} \text{ and } \Pi \leq (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}} \\ D + 1 + \frac{1+\beta}{\beta}(D + s + b(1-s))\Pi, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi \geq \frac{h}{b} \\ D + s + \frac{1+\beta}{\beta}(D + s)\Pi, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi < \frac{h}{b} \end{cases} \tag{17}$$

$$
Y = \begin{cases} Y_S + 1, & \text{for } \Pi \geq \Pi_{kink} \text{ and } \Pi \leq (\frac{b}{h})^{\frac{1}{\phi_\pi - 1}} \Pi_{kink}^{\frac{\phi_\pi}{\phi_\pi - 1}} \\ Y_S + 1, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi \geq \frac{h}{b} \\ Y_S + s, & \text{for } \Pi < \Pi_{kink} \text{ and } \Pi < \frac{h}{b} \end{cases} \tag{18}
$$

Graphically, the truncated AD and AS curves in equation (17) and equation (18) are in Figure 4 and Figure 5.[2] Notice that the AD curve becomes upward sloping when the economy enters into liquidity trap, since nominal interest rate no longer reacts to changes in inflation rate. The AD curve is also not continuous at the inflation level of $\frac{h}{b}$, since when inflation falls below this level, the risky projects will not be carried out. The AS curve is also not continuous at the inflation rate of $\frac{h}{b}$ for precisely the same reason.

Figure 4: Aggregate Demand Curve (Truncated)

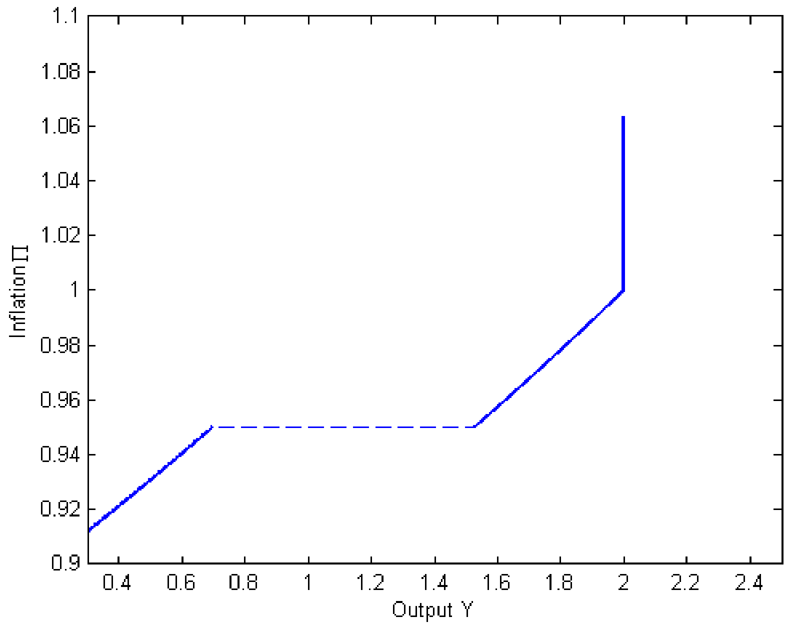


4 Liquidity Trap

We now analyze the equilibrium of the model. When the future safe income D and the amount of safe projects s are high enough, the risk in the economy is low. Lower risk makes the lender (middle-aged agent) more willing to lend, hence increases the spending of the borrower and increases aggregate demand. High aggregate demand ensures that

[2]The parameter choices for the graphs are: $\bar{L} = 1$, $\beta = 0.98$, $h = 0.095$, $b = 0.1$, $= 0.9$, $s = 0.17$, $D = 0.17$, $\alpha = 0.5$, $^* = 1.02$, $\phi_\pi = 2$, $i^* = r^f$. Here r_f is the real interest rate when the economy is producing at full capacity $Y = Y^f + 1$.

Figure 5: Aggregate Supply Curve (Truncated)



inflation is also high, so the risk-averse investor (middle-aged agent) is willing to invest in the risky projects operated by the young agent, which further increases aggregate demand. Graphically, When the future safe income D and the amount of safe projects s are high enough, we have the full employment equilibrium illustrated in Figure 6.[3] The equilibrium output level is $Y^f + 1$, which is the maximum possible output level for this economy.

However, when the future safe income D and the amount of safe projects s are low, the risk in the economy is high. Increased risk makes the lender (middle-aged agent) less willing to lend, hence reduces the spending of the borrowers and reduces aggregate demand. The fall in aggregate demand can pull the economy into a liquidity trap with low inflation. Faced with such a liquidity trap, the investor (middle-aged agent) will optimally choose to hold cash rather than investing in risky real projects operated by the young agent, which further deters aggregate demand. Therefore, the economy falls into a liquidity trap with low output, low investment and low inflation. Graphically, when the future safe income D and the amount of safe projects s are low, we have the liquidity trap equilibrium illustrated in Figure 7.[4]

[3] The parameters in the full employment equilibrium of Figure 6 are: $\bar{L} = 1$, $\beta = 0.98$, $h = 0.095$, $b = 0.1$, $= 0.9$, $s = 0.17$, $D = 0.17$, $\alpha = 0.5$, $\Pi^* = 1.02$, $\phi_\pi = 2$, $i^* = r^f$. The parameters are not calibrated and are chosen for illustrative purpose.

[4] The parameters in the liquidity trap equilibrium of Figure 6 are: $\bar{L} = 1$, $\beta = 0.98$, $h = 0.095$, $b = 0.1$, $= 0.9$, $s = 0.05$, $D = 0.05$, $\alpha = 0.5$, $\Pi^* = 1.02$, $\phi_\pi = 2$, $i^* = r^f$. The parameters are not calibrated and are chosen for illustrative purpose.

Figure 6: Full Employment Equilibrium

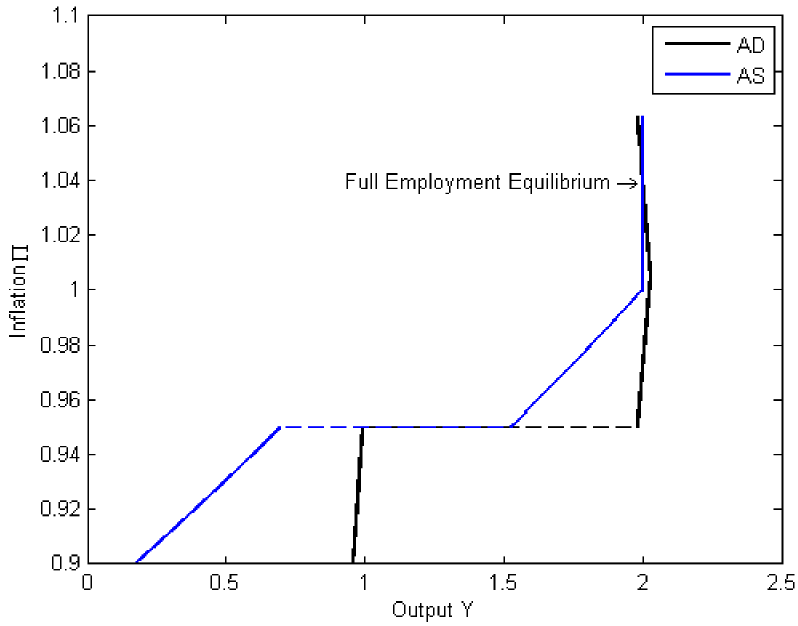
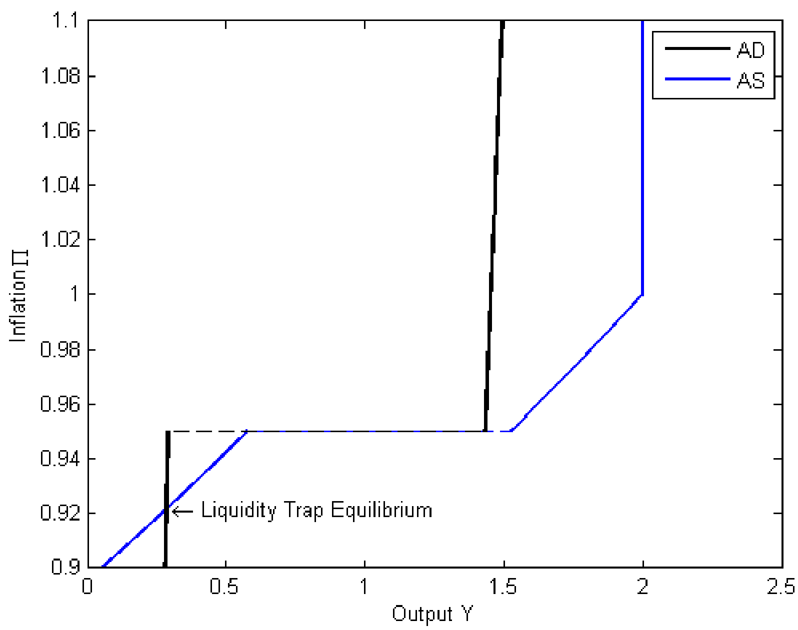


Figure 7: Liquidity Trap Equilibrium



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